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CONDITIONS AT DISCONTINUITIES IN POLARIZABLE MEDIA

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Conditions at discontinuity surfaces in polarizable media in the presence of an electric field are defined. It is shown with the use of the concept of evolution that in electrohydrodynamics the majority of gases with their characteristic linear dependence of permittivity on density suffer compression shocks known in conventional hydrodynamics.

1. Conditions at discontinuities. Let us consider a medium which in an electric field is polarizable according to the linear rule $\mathbf{D} = \varepsilon \mathbf{E}$, where the medium permittivity ε is a function of density and temperature. Let us formulate the conditions at the discontinuity surface in such media.

Let Σ be a surface bounding volume V which contains a certain part of the discontinuity surface S, $\mathbf{n}$ be a vector of the normal to surface Σ, L be the contour bounding surface Σ_1 defined by the intersection of volume V by a plane passing through the normal $\mathbf{n}$ and tangent τ to Σ at some point M.

We consider a stabilized motion. Using all fundamental equations of motion of a polarizable fluid in their integral form in approximations common to electrohydrodynamics and, passing to the limit by contracting V and Σ to S and S to some point M, we obtain at the discontinuity the following conditions:

$$\{\rho v_n\} = 0, \qquad \left\{\rho v_n + p + \frac{E^2}{8\pi}\left[\varepsilon - \rho\left(\frac{\partial\varepsilon}{\partial\rho}\right)_T\right] - \frac{\varepsilon E_n}{4\pi}\right\} = 0 \tag{1.1}$$

$$\left\{\rho v_n v_\tau - \frac{\varepsilon E_n E_\tau}{4\pi}\right\} = 0, \qquad \rho v_n\left\{\frac{v^2}{2} + w - \frac{E^2}{8\pi\rho}\left[T\left(\frac{\partial\varepsilon}{\partial T}\right)_\rho - \rho\left(\frac{\partial\varepsilon}{\partial\rho}\right)_T\right]\right\} = 0$$

$$\{D_n\} = 0, \qquad \{E_\tau\} = 0 \qquad (\{f\} = f_1 - f_2)$$

Let us condider two kinds of shocks: (a) a shock without flow of fluid through the discontinuity surface and (b) a shock through which the fluid flows.

a) If the normal velocity component at the discontinuity is zero, we have the following relationships:

arbitrary $\rho_{1,2}$ and $v_{\tau 1,2}$

$$\{p\} + \left\{\frac{E_n^2}{8\pi}\left[\left(\frac{\partial\varepsilon}{\partial\rho}\right)_T + \varepsilon\right]\right\} + \frac{E_\tau^2}{8\pi}\left\{\varepsilon - \rho\left(\frac{\partial\varepsilon}{\partial\rho}\right)_T\right\} = 0$$

Hence it is possible to obtain from the integral equation the relation between the jumps of pressure and of the electric field.

b) assuming that $v_n \neq 0$ and setting $v_{\tau 1} = 0$ which results in $v_{\tau 2} = 0$, we obtain

$$\{\rho v_n\} = 0 \tag{1.2}$$

$$\rho v_n \{v_n\} + \{p\} - \left\{\frac{E_n^2}{8\pi}\left[\rho\left(\frac{\partial\varepsilon}{\partial\rho}\right)_T + \varepsilon\right]\right\} + \frac{E_\tau^2}{8\pi}\left\{\varepsilon - \rho\left(\frac{\partial\varepsilon}{\partial\rho}\right)_T\right\}$$

$$\frac{v_n^2}{2}+w-\frac{E^2}{8\pi\rho}\left[T\left(\frac{\partial\varepsilon}{\partial T}\right)_\rho-\rho\left(\frac{\partial\varepsilon}{\partial\rho}\right)_T\right]$$
$$\{D_n\}=0, \qquad \{E_\tau\}=0$$

2. Equation of the shock adiabate and momenta in the case when permittivity depends only on density. Let us consider the case which is characteristic of the majority of gases, i.e. when permittivity is a linear function of density: $\varepsilon - 1 = \alpha\rho$ in a fairly wide range of temperatures.

We introduce the notation

$$P=\frac{p_2}{p_1},\quad V=\frac{\rho_1}{\rho_2};\quad e^2=\frac{E_{1n}^2}{8\pi p_1},\quad \rho v_n=m,\quad \alpha^*=\alpha\rho_1,\quad k=\frac{\gamma+1}{\gamma-1},\quad \Omega=\frac{m^2}{p_1\rho_1}$$

For a perfect gas, after some computations, we obtain from system (1.2) the equation for the shock adiabate

$$P=\frac{1}{kV-1}\left[k-V+\frac{e^2\alpha^{*2}(1-V)^3}{(\alpha^*+V)^2}\right],\qquad w=\frac{\gamma}{\gamma-1}\frac{p}{\rho} \tag{2.1}$$

This equation has two asymptotes

$$P=-(1+e^2\alpha^{*2})/k, \qquad V=1/k$$

This shows that in the interval $1/k<V<1$ the shock adiabate monotonically decreases with increasing V. Equations of the adiabate are shown in Fig. 1 in the form of curves drawn in solid lines.

From system (1.2) it is also possible to derive the equation of momenta

$$P=1+\Omega(1-V)-\frac{e^2\alpha^{*2}(1-V)(2\alpha^*+1+V)}{(\alpha^*+V)^2} \tag{2.2}$$

which has two asymptotes: $V=-\alpha^*$ and $P=-\infty$. It will be seen that

$$\partial P/\partial V=0 \quad \text{for} \quad V_{\max}=[2e^2\alpha^{*2}(\alpha^*+1)^2/\Omega^{1/3}]-\alpha^*$$

For a medium defined by parameters $\rho\sim10^{-3}$ g/cm³, $v\sim10^3$ cm/sec, and $E<30$ cgse units, we obtain $V_{\max}<1$. The curve of this equation of momenta is shown in Fig. 1. by a solid line which intersects the adiabate (2.1) at two points.

3. Evolutionary properties of electrohydrodynamic shock waves. It is shown in [1] that a discontinuity is stable if the number of weak perturbations radiating from it is smaller by one than the number of equations. Two kinds of weak perturbations are known in electrohydrodynamics [2, 3], namely: entropy waves which propagate together with the fluid and acoustic waves which propagate at the speed $a=\pm(a_0^2+\alpha^2\rho E^2/4\pi\varepsilon)^{1/2}$. If the velocity of these waves is directed away from the shock wave, we shall consider them as coming toward the discontinuity. For fixed D_n and E_τ the discontinuity is defined by three equations of conservation: of mass, momentum, and energy. Hence for the evolution to take place

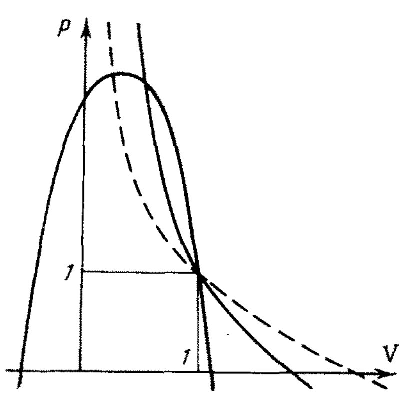

Fig. 1

it is necessary that two waves radiate from the discontinuity. This occurs when the velocity upstream of the discontinuity is higher than the speed of sound, while the velocity downstream of it is lower than the speed of sound, i.e. $v_1 > a_1$ and $v_2 < a_2$. The condition $v_1 > a_1$ implies that

$$v_1^2 > a_0^2 + \alpha^2\rho_1 E_1^2 / 4\pi\varepsilon \quad \text{or} \quad -\Omega + e^2\alpha^{*2} / (\alpha^* + 1) < -\gamma \tag{3.1}$$

The left-hand side of the last inequality contains a quantity which is the tangent of the slope of the curve which represents the equation of conservation of momenta, while that in the right-hand side is the tangent of the shock adiabate at point $V = 1$, $P = 1$. It follows from (3.1) that in the case of evolutionary waves the line corresponding to the equation of conservation of momenta at point $V = 1$, $P = 1$ for $V < 1$ lies above the shock adiabate and must always intersect the latter in the interval $1 / k < V < 1$. We have thus established that in electrohydrodynamics in the case of linear dependence of permittivity on density shock waves are always compression waves. The normal component of the elctric field downstream of the wave front is smaller than the normal component of that field upstream of the front.

It will be seen from (2.1) that for $V < 1$ the curve of the shock adiabate lies higher than the conventional gasdynamic adiabate, i.e. it is in region $S > S_1$, where S_1 is the entropy at point $P = 1$, $V = 1$. The increase of entropy at the shock also shows that the latter is a compression shock.

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ON THE STABILITY OF COUETTE FLOW OF A SECOND-ORDER FLUID

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The stability of a Couette flow of incompressible non-Newtonian second-order fluid at high Reynolds numbers [1] is considered within the limits of the linear theory of hydrodynamic stability. Unlike the Couette flow of a Newtonian (first-order) fluid which according to the linear theory is stable, the flow considered here may loose its stability even in the linear approximation.

The problem of hydrodynamic stability of simple flows of non-Newtonian fluids was considered in a fairly large numer of publications [2–4] in which the effect of elastic